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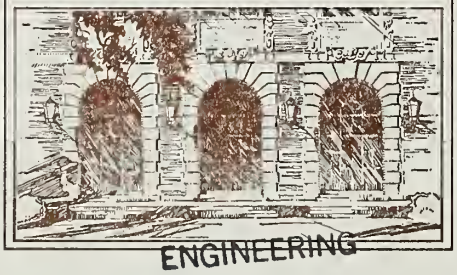

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TOWARD KEYSTONES FOR THEORIES OF NATURAL COMMUNICATIONS

by

Robert M. Ray III

August 1974

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ABSTRACT

Considering synthetic patterns as the most elementary units of semantic information, pattern encoding and decoding mechanisms are discussed as the basic components of semantic communication systems. Employing mathematical concepts long used by social scientists for modeling spatial interaction patterns within complex social systems, a particular representation of pattern information is presented and, with respect to this notation, a numerical measure of pattern correlation is developed that permits transducer recognition of internally stored patterns maximally correlating with patterns received from an external, noisy information source. Possible generalizations of the model lead to brief discussion of the potential utility of the concepts employed within the development of macro theories of natural communication processes.

Introduction

Attempting to formalize in both philosophical and mathematical terms the primacy of information flow within systems exhibiting purposive behavior, Wiener termed cybernetics the study of all communication and control processes, whether mechanical, biological, or social in nature.[1,2,3] Not independently of Wiener, Shannon established an outline adequate for conceptual analysis of any communication system and, distinguishing sharply between message semantics and message encodings, formulated a mathematical theory of transmission efficiency for communication channels linking message encoding and decoding operations.[4] Inspired by the utility and elegance of Shannon's concepts and mathematical formulations, Weaver (and others) immediately speculated that the contributions of Wiener and Shannon would generalize toward a more comprehensive theory of communications treating message semantics as well as transmission efficiencies.[5] Despite considerable development of Wiener-Shannon information theory to date,[6,7] as well as parallel developments in psychology, linguistics, and language philosophy,[8,9,10,11] there remains lacking any viable principle for integration of existing theory toward a conceptual framework of sufficient coherence to support hypotheses concerning semantic communications within natural systems.[12,13]

Employing Shannon's premise that "the fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point," below we revise Shannon's diagram of a general communication system to emphasize the nature

of the encoding and decoding operations that occur at either end of a communication channel. Messages conveyed from source to destination, however complex, are necessarily organized in terms of a finite vocabulary of semantic elements common to both encoding and decoding operations alike. Except for labels and schematic indications of information flow, the symmetry of the diagram again reflects the bi-directional nature of communications--processes occurring always in the presence of noise.

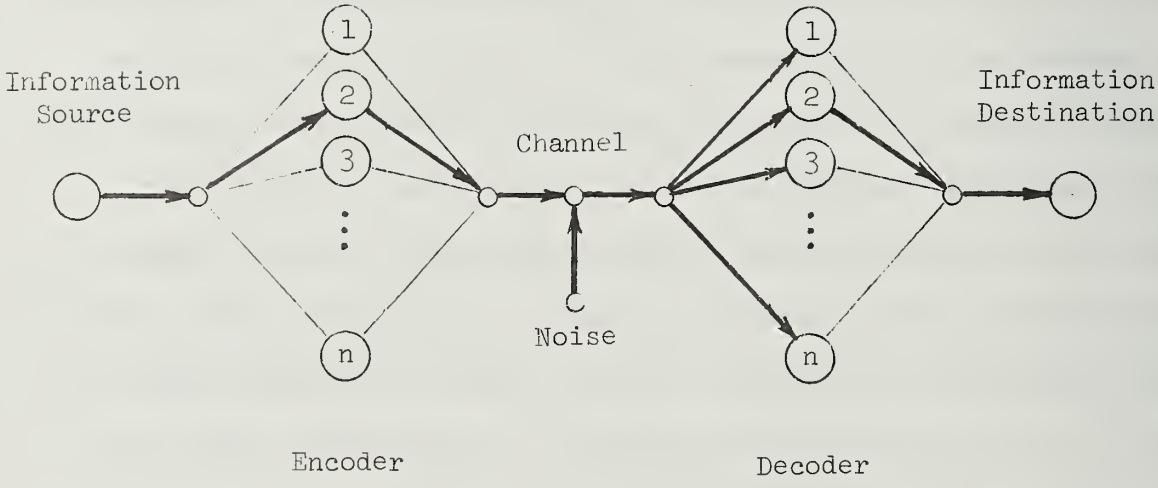

Figure 1. Shannon's schematic diagram of a general communication system.

If we use the term pattern generally to encompass all basic organizational units of semantic information (signs, symbols, characters, words, etc.), we may regard a fundamental goal of pattern information processing research to be the specification of formal models of the message encoding and decoding operations that enable semantic information

transmission between message source and destination. A primary theoretical problem of message encoding requires a formal representation of quantized semantic patterns appropriate for message organization and transmission. A corresponding decoding problem requires specification of models of pattern information recognition of sufficient efficacy and generality to support at least theoretical estimation of the degree of system complexity necessary for message comprehension. At a minimum level of complexity, the problem requires definition of measures of pattern information correlation, similarity, or association that may be employed to effect accurate transducer addressing of those patterns internal to its own vocabulary that best communicate the semantics of the information transmitted to it by an external message source.

The diversity of techniques currently employed as pattern recognition models makes difficult any simple typology of methodologies. (See references (14) and (15).)

For the purposes of this paper, it will be sufficient to distinguish only two broad approaches to pattern recognition which we will refer to as analytic and synthetic. By analytic, we mean any methodology for which pattern classification depends either upon analysis of component pattern parts, attributes, or features (e.g., statistical methods), or upon analysis of structural relationships between pattern components (e.g., syntactic methods). By synthetic, we mean simply any methodology that treats patterns as integral units of information and that categorizes patterns in accordance with similarities determined by methods of direct comparison (e.g., template matching techniques and Widrow's "rubber-masking" approach.[16,17])

The inadequacies of traditional correlation techniques for quantification of pattern similarities have discouraged greatly to date the research and development of pattern recognition models based on methods of direct comparison.[18,19,20] On the other hand, the rationale underlying the various analytic methodologies so widely adopted seems contrary to the basic notion of pattern gestalt--that the whole is more than the sum of its parts. The perplexities of feature extraction and selection processes within so many analytic models of recognition may well attest to the synthetic nature of pattern information in general. One purpose of this paper is to suggest that pattern information processing research may have abandoned too soon the search for quantitative measures of synthetic pattern similarity.

Relying greatly on mathematical concepts long employed by social scientists for modeling certain aspects of organization exhibited by systems of social interaction within community spatial structure, below we suggest a particular representation of patterns as spatial distributions of information and develop a mathematical index of pattern information correlation expressible for any two patterns so defined. The measure of pattern correlation is first developed geometrically for pictorial patterns as a coefficient of configuration similarity that may be estimated invariant with respect to individual pattern intensities, sizes, positions, and proximate orientations. This particular mathematical formulation of pattern correlation then suggests a general network model of pattern information processing that allows representation of pattern associations as network distributions of information. These generalizations

lead to consideration of the particular combination of concepts employed as a mathematical basis for macro theories of information processing within natural communication systems.

Patterns and Pattern Dissimilarities

While the generality surrounding the concept of a pattern makes difficult any single verbal definition, to assist the present mathematical discussion we offer the following: a pattern is a collection of quantized information distributed over a set of spatial elements characterizing some more complex information source. If we adopt at least provisionally the above definition, we may represent mathematically a particular pattern f as a partitioned array $(W|X)_f$, as tall as there are elements of f, where W is a matrix of coordinates indicating the relative spatial positions of all elements of f and X is a vector of positive reals indicating the proportional distribution of quantized units of information--pattern quits (quantization units)-- across all pattern elements.

For mathematical convenience we will assume normalization of pattern intensity so that $\sum_i x_i = 1$. Thus X may be considered a discrete probability distribution of pattern quits over pattern elements. Also, we will assume normalization of patterns with respect to positions and sizes so that pattern centroids are coincident at a common origin, $\sum_i x_i \, w_{i,k} = 0$ for each spatial dimension k, and second moments about the origin equal unity, $\sum_i x_i \sum_k w_{i,k}^2 = 1$.

With such mathematical notation we consider the following geometric pattern recognition problem: given a set of normalized patterns F, determine a symmetric non-negative scalar index of pattern dissimilarity, $\Delta_{f,g} = \delta[(W|X)_f, (Y|Z)_g]$, pairwise computable for all $f \varepsilon F$ and $g \varepsilon F$, such that

$\Delta_{f,g}$ approaches zero as the similarity of f and g increases. Below we develop such a criterion of pattern dissimilarity that is by normalization invariant with respect to individual pattern intensities, sizes, and positions and by computation optimal with respect to small displacements of individual pattern orientations.

For any two normalized patterns f and g we establish an index of pattern dissimilarity in the following manner. We determine simultaneously a weighted correspondence of elements between f and g and a registration of f with respect to g such that there exists maximal spatial congruence of corresponding elements between f and g. We then take as our criterion of pattern dissimilarity the weighted sum of squared distances between all pairs of elements corresponding between f and g where the weights of the sum reflect the degree of correspondence between each element pair.

Let the two patterns f and g consist of m and n elements respectively and let their normalized representations be denoted $(W|X)_f$ and $(Y|Z)_g$. We represent a particular weighted correspondence of elements between f and g as a matrix Q ($m \times n$) satisfying

$$(1) \quad \sum_{i}^{m} q_{i,j} = z_j \qquad j = 1, \ldots, n$$

$$(2) \quad \sum_{j}^{n} q_{i,j} = x_i \qquad i = 1, \ldots, m$$

$$(3) \quad q_{i,j} \geq 0 \qquad i = 1, \ldots, m$$

$$j = 1, \ldots, n.$$

Let $\pi_{f,g}$ denote the set of all Q matrices satisfying (1), (2), and (3) for given X of f and Z of g. Now by normalization of pattern intensities $\sum_i x_i = \sum_j z_j = 1$, hence $\sum_i \sum_j q_{i,j} = 1$. Since also $q_{i,j} \geq 0$ for all i and j,

we may consider any $Q \varepsilon \pi_{f,g}$ a discrete bivariate probability density function characterizing a joint distribution of corresponding quits over the elements of f and the elements of g.

Let a particular spatial registration of f with respect to g be denoted $\sigma WR + JT'$ where J is the vector $(1, \ldots, 1_m)'$, T is a translation vector, σ is a scale factor, and R is a rotational transformation. For a given registration of f with respect to g, let S $(m \times n)$ be the resulting matrix of squared Euclidean distances between the elements of f and the elements of g.

$$(4) \quad s_{i,j} = \sum_k [\, \sigma(\sum_l w_{i,l}\, r_{l,k} - t_k) - y_{j,k}]^2 \qquad \begin{matrix} i = 1,\ldots,m \\ j = 1,\ldots,n \end{matrix}$$

Let $\sum_{f,g}$ denote the set of S matrices obtainable for f and g by (4) over all positive scalars σ, all translations T, and all legitimate rotations R.

Our criterion of pattern dissimilarity may then be formulated:

$$(5) \quad \Delta_{f,g} = \min_{Q\varepsilon\pi_{f,g},\ S\varepsilon\sum_{f,g}} \sum_i^m \sum_j^n q_{i,j}\, s_{i,j} = \min_{Q\varepsilon\pi_{f,g},\ S\varepsilon\sum_{f,g}} \operatorname{tr}(Q'S).$$

Note that such an index of pattern dissimilarity may be interpreted as a minimal mean squared error of registration between all corresponding pattern quits of f and g.

Now for Q given, consider all possible translations of f with respect to g and write

$$(6) \quad \Delta = \sum_i^m \sum_j^n q_{i,j} \sum_k [(w_{i,k} - t_k) - y_{j,k}]^2.$$

To determine the particular T that minimizes Δ over all translations of f with respect to g, differentiate (6) with respect to t_k to obtain

$$(7)\quad \frac{d\Delta}{dt_k} = \sum_i^m \sum_j^n q_{i,j} \sum_k (2t_k - 2w_{i,k} + 2y_{j,k})$$

from which it may be determined that $t_k = 0$ where $\sum_i \sum_j q_{i,j}\, w_{i,k} = \sum_i \sum_j q_{i,j}\, y_{j,k}$, or where $\sum_i x_i\, w_{i,k} = \sum_j z_j\, y_{j,k}$. This implies that an optimal registration between f and g requires a coincidence of pattern centroids. Since by normalization pattern centroids already occur at a common origin and since these centroids remain invariant over pattern rotations and scale changes, we may conclude that no further consideration of pattern translations is necessary in minimizing $\Delta_{f,g}$.

Now consider all possible positive scale factors σ applied to f so that

$$(8)\quad \Delta = \sum_i^m \sum_j^n q_{i,j} \sum_k (\sigma\, w_{i,k} - y_{j,k})^2.$$

Differentiating Δ with respect to σ we find that the particular σ minimizing (8) is given by

$$(9)\quad \sigma = \Big[\sum_i^m \sum_j^n q_{i,j} \sum_k w_{i,k}\, y_{j,k}\Big] \Big/ \Big[\sum_i^m \sum_j^n q_{i,j} \sum_k w_{i,k}^2\Big].$$

If σ scales optimally W with respect to Y, then by symmetry, σ^{-1} scales optimally Y with respect to W. By an identical analysis we may determine for σ^{-1} the expression

$$(10)\quad \sigma^{-1} = \Big[\sum_i^m \sum_j^n q_{i,j} \sum y_{j,k}^2\Big] \Big/ \Big[\sum_i^m \sum_j^n q_{i,j} \sum_k w_{i,k}\, y_{j,k}\Big].$$

Hence $\sigma = \sigma^{-1} = 1$ where $\sum_i \sum_j q_{i,j} \sum_k w_{i,k}^2 = \sum_i \sum_j q_{i,j} \sum_k y_{j,k}^2$, or where $\sum_i x_i \sum_k w_{i,k}^2 = \sum_j z_j \sum_k y_{j,k}^2$. Again, by normalization this condition is met, since pattern second moments about the origin are set to unity. Hence, in minimizing $\Delta_{f,g}$ we may also exclude all further consideration of pattern scales.

The foregoing analysis demonstrates that the pattern positions and sizes yielded by normalization are optimal with respect to the minimal $\Delta_{f,g}$ independent of whatever correspondences Q are defined between the elements of f and g and independent of whatever rotation R may be chosen to effect maximum spatial congruence of corresponding elements of f and g. It remains to determine simultaneously an optimal correspondence matrix Q and a rotation R of f yielding an optimal registration of f and g such that the minimal $\Delta_{f,g}$ may be determined.

In the following sections we present a computational solution to (5) via iterative solution two interdependent subproblems--the correspondences problem, which requires minimization of $\Delta_{f,g}$ over all $Q \varepsilon \pi_{f,g}$ for fixed S, and the transformation problem, which requires determination of a rotation R that minimizes $\Delta_{f,g}$ over all $S \varepsilon \sum_{f,g}$ for fixed Q.

The Correspondences Problem

Consider f and g as two patterns taken from a set of quantized patterns that may be assumed to be normalized with respect to intensity, position, and size and standardized with respect to orientation, e.g., a font recognition problem. Since in such a case we may assume no rotational freedom for either pattern, we may consider $\sum_{f,g}$ to consist of a single S matrix determined directly by

$$(11)\quad s_{i,j} = \sum_{k} (w_{i,k} - y_{j,k})^2 \qquad \begin{matrix} i = 1, \ldots, m \\ j = 1, \ldots, n. \end{matrix}$$

In such an instance, our formulation of pattern dissimilarity reduces to

$$(12)\quad \Delta_{f,g} = \min_{Q \varepsilon \pi_{f,g}} \sum_{i}^{m} \sum_{j}^{n} q_{i,j}\, s_{i,j} = \min_{Q \varepsilon \pi_{f,g}} \operatorname{tr}(Q'S)$$

where again $\pi_{f,g}$ is the set of all Q matrices satisfying (1), (2), and (3).

The optimization problem given by (1), (2), (3), and (12), where $\sum_i x_i = \sum_j z_j = \gamma$ but γ not necessarily unity, may be recognized as the Hitchcock or transportation problem of linear programming.[21,22,23]

Typically, the problem requires determination of a matching between a spatially distributed set of economic supplies and a spatially distributed set of demands such that the total cost of all material movements from suppliers to buyers is minimal. For such problems, computational algorithms are well known and solution properties well documented. While any of these algorithms may be used for computation of $\Delta_{f,g}$ in the present case, the characteristics and computational requirements of pattern information processing in general compel us to look further for mathematical techniques more appropriate for parallel processing systems.

A related problem known to transportation planners is called the entropy network distribution problem.[24,25] The problem arises where it is desirable to simulate traffic flows within a metropolitan region given data describing distributions of populations and economic activities over some set of analysis zones subdividing the region, zone-to-zone travel times, and estimates of mean travel times for specific

types of trips within the region. Borrowing the notation of our pattern correspondences problem, let Δ represent the mean travel time for all home-work commuting trips, let X be the probability distribution of workers over m residential zones, let Z be the distribution of jobs over n employment zones, and let S be a matrix of network travel times between any residential zone and any employment zone. The problem requires determination of a most probable, mean, or maximum entropy joint probability distribution Q with marginals X and Z such that each element $q_{i,j}$ represents the forecasted proportion of all trips occurring between the i-th residential zone and the j-th employment zone. Mathematically, the problem is formulated

$$(13) \quad \max H = - \sum_{i}^{m} \sum_{j}^{n} q_{i,j} \log q_{i,j}$$

subject to (1), (2), and (3) and the additional mean travel time constraint

$$(14) \quad \sum_{i}^{m} \sum_{j}^{n} q_{i,j} \, s_{i,j} = \Delta.$$

Note that constraint (14) may be considered simply an a priori specification of overall network distribution efficiency or energy expenditure.

The solution to the problem is given by

$$(15) \quad q_{i,j} = x_i \, u_i \, z_j \, v_j \exp(-\beta \, s_{i,j}) \qquad \begin{matrix} i = 1, \ldots, m \\ j = 1, \ldots, n \end{matrix}$$

where β represents the Lagrange multiplier associated with constraint (14) and the u_i and v_j are functions of the Lagrange multipliers associated with constraint sets (1) and (2). It has been shown [26] that corresponding

to any real β there exists a unique Q maximizing (13) and satisfying (1), (2), and (3) given by (15) where the parameters u_i and v_j may be determined by iterative solution of the equations

$$(16)\quad u_i = [\sum_j^n z_j\, v_j \exp(-\beta\, s_{i,j})]^{-1} \qquad i = 1, \ldots, m$$

$$(17)\quad v_j = [\sum_i^m x_i\, u_i \exp(-\beta\, s_{i,j})]^{-1} \qquad j = 1, \ldots, n.$$

Additionally, it has been shown that there exists a monotonic mapping between all β and all feasible Δ such that as β approaches $-\infty$, Δ approaches Δ_{max}, and as β approaches $+\infty$, Δ approaches Δ_{min}, where Δ_{max} and Δ_{min} respectively denote the maximum and minimum values of Δ possible for given S, X, and Z. [27,28] Together these results yield theoretical justification for iterative determination of the unique Q maximizing (13) and satisfying the network distribution efficiency constraint (14) as well as constraints (1), (2), and (3). Since Δ_{min} of the entropy network distribution problem is analogous to the pattern dissimilarity criterion of our present pattern recognition problem, these results also imply that we may at least theoretically formulate a maximum entropy correspondence matrix Q*, unique and optimal with respect to the minimal $\Delta_{f,g}$, via equations (15), (16), and (17) with the parameter β set to ∞.

Using theorems developed elsewhere [29] and well known properties of the Hitchcock model, Evans [28] demonstrates several features of the matrix Q* and suggests a strategy by which it may be computed. Let E denote a binary matrix such that $e_{i,j} = 1$ for all subscript pairs (i,j) where $q^* > 0$ and $e_{i,j} = 0$ elsewhere. (Properties of the Hitchcock model imply that E will be sparse). The desired Q* and the matrix E then interrelate arithmetically in the form

$$(18)\quad q^{*}_{i,j} = x_i\, u^{*}_i\, z_j\, v^{*}_j\, e_{i,j}$$

where the u^{*}_i and v^{*}_j satisfy the relations

$$(19)\quad u^{*}_i = [\sum_{j}^{n} z_j\, v^{*}_j\, e_{i,j}]^{-1} \qquad i = 1, \ldots, m$$

$$(20)\quad v^{*}_j = [\sum_{i}^{m} x_i\, u^{*}_i\, e_{i,j}]^{-1} \qquad j = 1, \ldots, n.$$

Evans' method for exposing E, and hence Q*, requires initially a solution to the Hitchcock problem (12) determined presumably by traditional methods.

If for our particular entropy network problem we may assume that convergence of (15), (16), and (17) is well-conditioned for large β, then a matrix Q* may be computed heuristically in the following manner. Establish the matrix $\hat{E}$ such that $\hat{e}_{i,j} = \exp(-\beta\, \hat{s}_{i,j})$ where $\hat{S}$ is S scaled linearly to have elements within a specified interval and β is chosen as large as computational considerations permit. Determine estimates of U* and V* via iterative solution of

$$(21)\quad \hat{u}_i = [\sum_{j}^{n} z_j\, \hat{v}_j\, \hat{e}_{i,j}]^{-1} \qquad i = 1, \ldots, m$$

$$(22)\quad \hat{v}_j = [\sum_{i}^{m} x_i\, \hat{u}_i\, \hat{e}_{i,j}]^{-1} \qquad j = 1, \ldots, n$$

and then estimate Q* via

$$(23)\quad \hat{q}_{i,j} = x_i\, \hat{u}_i\, z_j\, \hat{v}_j\, \hat{e}_{i,j} \qquad i = 1, \ldots, m \qquad j = 1, \ldots, n.$$

Now assuming that β is sufficiently large such that $\hat{Q}$ is close to Q*, then we may expect small elements of $\hat{Q}$ to correspond to zero elements of Q*. Hence we may select for each $q^{*}_{i,j}$ some threshhold value,

say $\bar{q}_{i,j} = x_i * z_j$, and approach E by resetting $\hat{e}_{i,j} = 0$ whenever $\hat{q}_{i,j} < \bar{q}$ for any particular solution to (21), (22), and (23); that is,

$$(24)\quad \hat{e}_{i,j} = \left\{ \begin{array}{r} \exp(-\beta \hat{s}_{i,j}) \text{ if } \hat{q}_{i,j} \geq \bar{q}_{i,j} \\ 0 \text{ if } \hat{q}_{i,j} < \bar{q}_{i,j} \end{array} \right\} \begin{array}{l} i = 1, \ldots, m \\ j = 1, \ldots, n. \end{array}$$

Note that resetting $\hat{e}_{i,j} = 0$ at any point is equivalent to the assumption that pattern elements i and j have been determined non-corresponding and thus $q^*_{i,j} = 0$. Since initially $\hat{e}_{i,j}$ is taken always as $\exp(-\beta \hat{s}_{i,j})$, this assumption is also equivalent to the condition that $\hat{s}_{i,j}$ is large relative to other elements of $\hat{S}$, hence $\hat{e}_{i,j}$ approaches zero faster than other elements of $\hat{E}$ as β approaches ∞.

Iterative solution of (21), (22), (23), and (24) in the above manner should lead to stable values for $\hat{Q}$ and $\hat{E}$. At convergence, we may equate heuristically $\hat{Q}$ and Q^*. Setting all remaining positive elements of $\hat{E}$ to 1 we have a similar approximation of E. Q^* and E should be related via (18), (19), and (20).

We have gained some evidence that such a procedure is sufficient for determination of optimal pattern element correspondences, at least where elements of S are taken as squared Euclidean planar distances and m and n are small. While there is the possibility that the above procedure will prove ineffective (or inefficient) in other cases, the properties of the Hitchcock and entropy network models offer much promise for the development of parallel algorithms practical for specific applications. While we have indicated a possible course for the development of such algorithms, the primary intent of this section has been simply to introduce the conceptual utility of the entropy network model to pattern information processing research.

In this context, it should also be noted that minimum-distance classification of patterns using the distance criterion(5) does not necessarily require computation of the limiting Q^*. For most applications, we have found that a Q matrix determined directly via equations (15), (16) and (17), with β set to some large positive integer, yields approximate values of pattern dissimilarities via (5) completely satisfactory for recognition purposes.

The Transformation Problem

Now assume f and g taken from a set of patterns normalized with respect to intensity, position, and size but not standardized with respect to orientation. On the contrary, in this section we will assume given quantizations of f and g, again $(W|X)$ and $(Y|Z)_g$, to include small proper rotational displacements of individual pattern orientations that must be determined and accounted for in the computation of $\Delta_{f,g}$ via (5). Therefore, in comparing f and g, our problem is to determine a rotation $R \in \Theta$, the set of all proper rotations, of one pattern with respect to the other, say f with respect to g, yielding an S via (4) such that $\Delta_{f,g}$ via (5) is minimal over all $S \in \Sigma_{f,g}$ as well as all $Q \in \pi_{f,g}$. This transformation problem may be solved in the following manner.

Assuming small displacements of orientations for f and g, we may determine an initial correspondences matrix Q via the method just presented using provisionally for S the value given by (11). Then with Q fixed, our new estimate of pattern dissimilarity may be formulated

$$\Delta = \min_{S \in \Sigma_{f,g}} \operatorname{tr}(Q'S) \tag{25}$$

where our problem is again to determine a rotation **R** yielding a new estimate of S via (4) stepwise optimal with respect to Q.

Define $\underline{Q}$ and $\underline{S}$ as mn × mn diagonals such that

$$(26) \qquad (\underline{q}_{1,1}, \underline{q}_{2,2} \cdots, \underline{q}_{mn,mn}) = (q_{1,1}, q_{1,2}, \cdots, q_{m,n})$$

and

$$(27) \qquad (\underline{s}_{1,1}, \underline{s}_{2,2}, \cdots, \underline{s}_{mn,\,mn}) = (s_{1,1}, s_{1,2}, \cdots, s_{m,n})$$

then we may express (25) alternatively as

$$(28) \qquad \Delta = \min_{R \in \Theta} \operatorname{tr}(\underline{Q}'\underline{S}) = \operatorname{tr}(\underline{Q}\underline{S})$$

where the elements of $\underline{S}$ remain to be determined as a function of the unknown R.

Also define $\underline{W}$ to be mn × k where the first row of W is repeated n times as the first n rows of $\underline{W}$, the second row of W repeated as the next n rows of $\underline{W}$ and so forth. Define $\underline{Y}$ to be mn × k where the entire matrix Y is simply repeated vertically m times.

Now note that ordered diagonal elements of the matrix $[(\underline{W}R-\underline{Y})(\underline{W}R-\underline{Y})']$ are identically equal to the elements of $\underline{S}$, hence we may write

$$(29) \qquad \operatorname{tr}[(\underline{W}R-\underline{Y})(\underline{W}R-\underline{Y})'] = \operatorname{tr}(\underline{S})$$

and since $\underline{Q}$ is also diagonal we may restate (28) as

$$(30) \qquad \Delta = \min_{R \in \Theta} \operatorname{tr}[\underline{Q}(\underline{W}R-\underline{Y})(\underline{W}R-\underline{Y})'].$$

Defining $\underline{Q}^{\frac{1}{2}}$, such that $\underline{Q}^{\frac{1}{2}}\underline{Q}^{\frac{1}{2}} = \underline{Q}$, we may write

$$(31) \qquad \Delta = \min_{R \in \Theta} \operatorname{tr}[\underline{Q}^{\frac{1}{2}}(\underline{W}R-\underline{Y})(\underline{W}R-\underline{Y})'\underline{Q}^{\frac{1}{2}}]$$

and after manipulation,

$$\Delta = \min_{R\varepsilon\Theta} \operatorname{tr}\left[(\underset{\sim}{Q}^{\frac{1}{2}}\underset{\sim}{W}R-\underset{\sim}{Q}^{\frac{1}{2}}\underset{\sim}{Y})'(\underset{\sim}{Q}^{\frac{1}{2}}\underset{\sim}{W}R-\underset{\sim}{Q}^{\frac{1}{2}}\underset{\sim}{Y})\right]. \tag{32}$$

Now substitute $\tilde{\underset{\sim}{W}} = \underset{\sim}{Q}^{\frac{1}{2}}\underset{\sim}{W}$ and $\tilde{\underset{\sim}{Y}} = \underset{\sim}{Q}^{\frac{1}{2}}\underset{\sim}{Y}$ into (32) to obtain

$$\Delta = \min_{R\varepsilon\Theta} \operatorname{tr}\left[(\tilde{\underset{\sim}{W}}R-\tilde{\underset{\sim}{Y}})'(\tilde{\underset{\sim}{W}}R-\tilde{\underset{\sim}{Y}})\right] \tag{33}$$

which may be written

$$\Delta = \min_{R\varepsilon\Theta} \operatorname{tr}\left(R'\tilde{\underset{\sim}{W}}'\tilde{\underset{\sim}{W}}R - 2R'\tilde{\underset{\sim}{W}}'\tilde{\underset{\sim}{Y}} + \tilde{\underset{\sim}{Y}}'\tilde{\underset{\sim}{Y}}\right) \tag{34}$$

or, since the trace of a sum equals the sum of the traces,

$$\Delta = \min_{R\varepsilon\Theta} \left[\operatorname{tr}(R'\tilde{\underset{\sim}{W}}'\tilde{\underset{\sim}{W}}R) - 2\operatorname{tr}(R'\tilde{\underset{\sim}{W}}'\tilde{\underset{\sim}{Y}}) + \operatorname{tr}(\tilde{\underset{\sim}{Y}}'\tilde{\underset{\sim}{Y}})\right]. \tag{35}$$

Now consider the first and last terms of (35). Clearly both are independent of R. Furthermore, by normalization and our definitions of $\underset{\sim}{W}$, $\underset{\sim}{Y}$, $\tilde{\underset{\sim}{W}}$ and $\tilde{\underset{\sim}{Y}}$ we may write

$$\operatorname{tr}(R'\tilde{\underset{\sim}{W}}'\tilde{\underset{\sim}{W}}R) = \operatorname{tr}(\tilde{\underset{\sim}{W}}'\tilde{\underset{\sim}{W}}) = \operatorname{tr}\left[(\underset{\sim}{Q}^{\frac{1}{2}}\underset{\sim}{W})'(\underset{\sim}{Q}^{\frac{1}{2}}\underset{\sim}{W})\right] = \sum_i \sum_j q_{i,j} \sum_k w_{i,k}^2 = \tag{36}$$

$$\sum_i x_i \sum_k w_{i,k}^2 = 1$$

and

$$\operatorname{tr}(\tilde{\underset{\sim}{Y}}'\tilde{\underset{\sim}{Y}}) = \operatorname{tr}\left[(\underset{\sim}{Q}^{\frac{1}{2}}\underset{\sim}{Y})'(\underset{\sim}{Q}^{\frac{1}{2}}\underset{\sim}{Y})\right] = \sum_i \sum_j q_{i,j} \sum_k y_{j,k}^2 = \sum_j z_j \sum_k y_{j,k}^2 = 1. \tag{37}$$

Since also the middle term of (35) may be written $-2\mathrm{tr}[R'(\underline{Q}^{\frac{1}{2}}\underline{W})'(\underline{Q}^{\frac{1}{2}}\underline{Y})] = -2\mathrm{tr}(R'\underline{W}'\underline{Q}\underline{Y})$, we have shown that the transformation problem may be formulated equivalently as

$$(38)\quad \Delta = 2 - 2 \max_{R\epsilon\Theta} \mathrm{tr}(R'\underline{W}'\underline{Q}\underline{Y})$$

or,

$$(39)\quad \Delta = 2 - 2 \max_{R\epsilon\Theta} \mathrm{tr}(R'W'QY).$$

With reference to (39) we notice that solution of (5) is equivalent to solution of

$$(40)\quad P_{f,g} = \max_{R\epsilon\Theta,\ Q\epsilon\pi_{f,g}} \mathrm{tr}\,(R'W'QY)$$

where there exist the inverse monotonic mappings $\Delta_{f,g} = 2 - 2P_{f,g}$ and $P_{f,g} = 1 - \frac{1}{2}\Delta_{f,g}$. Since $\Delta_{f,g}$ is formulated as a mean sum of squared distances, its lower bound is zero hence, the upper bound for $P_{f,g}$ is unity. We may thus refer to $P_{f,g}$ as the pattern correlation of f and g and solve (40) as an alternative to (5).

Now if we allow R to be any orthogonal transformation, i.e., either a proper or improper rotation, the optimization problem given by any of the above formulations of our pattern transformation problem may be recognized as the Procrustes problem of psychometrics. (30,31) The problem arises in factor analysis and multi-dimensional scaling applications where it is desirable to compare two sets of factor coordinates independently determined for the same set of variables by rotation of one set to maximum spatial

congruence with the other to maximize between-set factor correlations. Mathematically, the problem is closely related to the canonical correlation problem of multivariate analysis.

It has been shown then that for our present problem where W and Y and hence $\underline{W}$, $\underline{Y}$, $\tilde{\underline{W}}$ and $\tilde{\underline{Y}}$ are of full-column rank k, an optimal orthogonal transformation R is given by

$$(41) \quad R = (HL^{-\frac{1}{2}}H')\,\tilde{\underline{W}}'\tilde{\underline{Y}}$$

where H (k × k) and L (k × k diagonal) represent respectively the eigenvectors and the eigenroots of the matrix $(\tilde{W}'\tilde{Y}\tilde{Y}'\tilde{W})$. (32) Since both $\tilde{\underline{W}}$ and $\tilde{\underline{Y}}$ are of rank k, all roots of $(\tilde{\underline{W}}'\tilde{\underline{Y}}\tilde{\underline{Y}}'\tilde{\underline{W}})$ are positive and we may take as the elements of $L^{-\frac{1}{2}}$ the reciprocals of the positive square roots of the elements of L. (30)

While it is true that computation of R via (41) optimizes Δ over all orthogonal transformations, definition of element correspondences Q via (11) (or via (4) where R has been chosen as a proper rotation) makes it extremely improbable that the maximum of tr(R'W'QY) will now occur for an improper rotation--that is, a reflection of f about some axis as well as a proper rotation of f with respect to g. Exceptions to this rule occur when comparing patterns whose coordinate matrices, W or Y, are only weakly of full-column rank, i.e. patterns of nominal full-column rank k whose spatial geometries can be accomodated with only slight distortion in a subspace of dimensionality $k' < k$. For example, where two patterns being compared represent quantized left and right parentheses, "(" and ")", and where Q has been given by (11), we would expect the R given by (41) to contain a horizontal reflection. On the other hand, if the two patterns being

compared are "M" and "W", both patterns strongly two-dimensional, we would not expect an R computed via the same method to contain a vertical reflection. In any case, where pattern reflections are significant, det R may be computed to detect improper rotations and further action may be undertaken appropriate to the specific application.

The Procrustes formulation of our pattern transformation problem is interesting, since it provides a general solution applicable for comparison of patterns of any dimensionality. In the case of planar pictorial patterns, however, the problem may be resolved in a more direct fashion.

Restricting R to be a proper rotation, let C = (W'QY) for a given Q and write max tr(R'W'QY) in (39) as

$$(42)\quad f(\alpha) = \max_{\alpha} \operatorname{tr} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} c_{1,1} & c_{1,2} \\ c_{2,1} & c_{2,2} \end{bmatrix}$$

or, equivalently,

$$(43)\quad f(\alpha) = \max_{\alpha} \; (c_{1,2} - c_{2,1}) \sin\alpha + (c_{1,1} + c_{2,2}) \cos\alpha \; .$$

Then let $A = (c_{1,2} - c_{2,1})$ and $B = (c_{1,1} + c_{2,2})$ and write (43) as

$$(44)\quad f(\alpha) = \max_{\alpha} \; (A \sin\alpha + B \cos\alpha).$$

Also, let $K = (A^2 + B^2)^{\frac{1}{2}}$ so that $A = K \sin\phi$ and $B = K \cos\phi$ and

$$(45)\quad f(\alpha) = \max_{\alpha} \; (K \sin\phi \sin\alpha + K \cos\phi \cos\alpha) = \max [K \cos(\alpha - \phi)].$$

The maximum occurs when $\alpha = \phi$ and is equal to $K = [(c_{1,2} - c_{2,1})^2 + (c_{1,1} + c_{2,2})^2]^{\frac{1}{2}}$.

The proper rotation maximizing (39) is then determined by the relations $\sin \alpha = A/K = (c_{1,2} - c_{2,1})/K$ and $\cos \alpha = B/K = (c_{1,1} + c_{2,2})/K$. Therefore, in comparing any two planar patterns f and g, a proper rotation R, stepwise optimal with respect to a given correspondence matrix Q, can always be determined directly as a function of the four elements of the matrix $C = (W'QY)$.

In the last section, we presented a method for determining Q optimal with respect to an assumed S. In this section, we have shown how an optimal transformation R, and hence an optimal S, may be determined with respect to a given Q. Since both subproblems are formulated to optimize the same criterion $\Delta_{f,g}$ (or $P_{f,g}$), iterative solution of both yields a value of $\Delta_{f,g}$ optimal at least locally over all $S \varepsilon \sum_{f,g}$ and all $Q \varepsilon \pi_{f,g}$. Thus given a set of quantized patterns G for which rotational displacements can be assumed small, a numerically expressible procedure exists for determining $\Delta_{f,g}$ for all $f \varepsilon G$ and all $g \varepsilon G$.

The Present Context of Developing Communications Theory

The particular approach to pattern communication systems presented above results from a cross-fertilization of two separate applied mathematics research efforts conducted by the author in the areas of metropolitan land use-transportation systems modeling and computer-assisted earth resources image analysis. Such a cross-fertilization of seemingly divergent efforts has been made possible largely by the research environment provided by the Center for Advanced Computation (CAC) of the University of Illinois at Urbana-Champaign--a widely diversified interdisciplinary team of scientists and technicians seeking appropriate applications of emerging

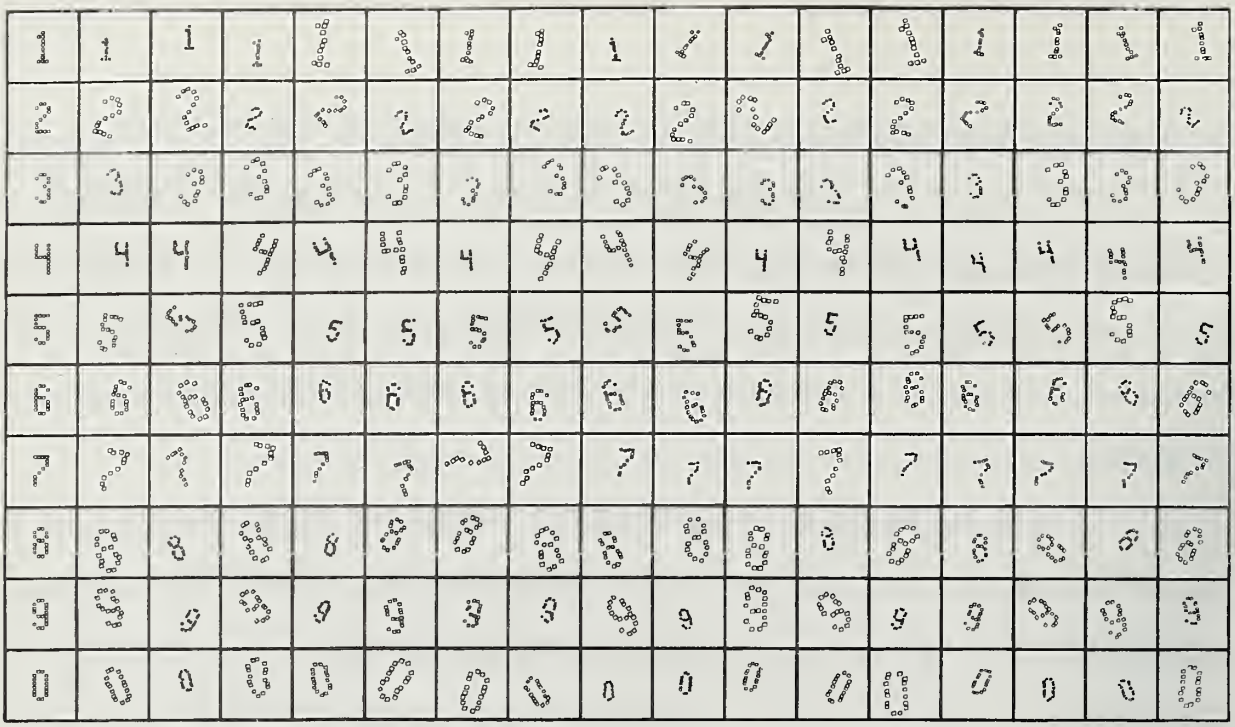

Figure 2. To evaluate the above numerical procedures, sixteen noisy copies of ten prototype patterns (extreme left) were generated. Despite considerable rotational displacements, all 160 patterns were correctly classified using a Fortran implementation of the above techniques.

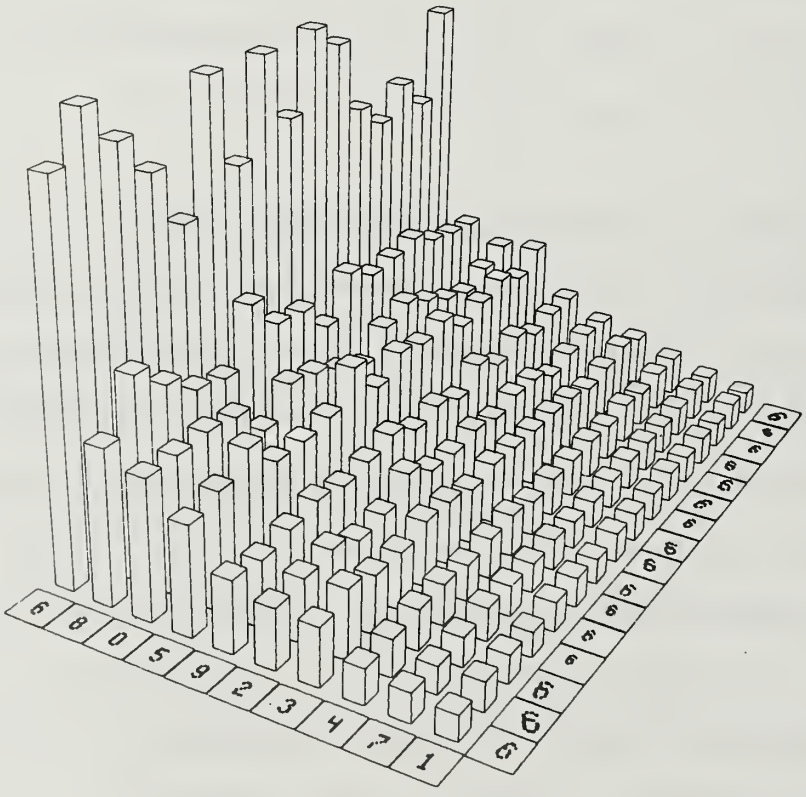

Figure 3. A computer graphic showing the rank order of prototype-to-pattern dissimilarity measures computed for the sixteen noisy "6's" of Figure 2. Individual blocks have been plotted proportional to $1/\Delta_{f,g}$. Also, prototypes have been ordered from left to right in accordance with overall prototype similarity with all example patterns depicted.

information processing and communication technologies to problems of social concern.

That communication systems, especially in the form of networks of transportation facilities, represent the prime determinants of regional and urban development patterns has long been emphasized by geographers and spatial economists. Since Wiener and Shannon, moreover, communication theory concepts have been embraced enthusiastically by urban analysts, both for the construction of descriptive theories of communities as evolutionary socioeconomic spatial systems and for the development of normative planning methodologies through which society may actively influence the determination of its future environments.

Likening the various forms of social interaction that bind together the community to the bonding forces that organize atoms within molecules, molecules within cells, and cells within organisms, Meier has proposed description of the community as a complex, self-organizing open system whose viability depends throughout on a hierarchy of vital processes struggling to conserve negative entropy.(33) Emphasizing the global impact of twentieth-century transportation and communication technologies, and the socioeconomic specialization that they afford, Webber has argued that our communities can now only be viewed as cultural clusters of individuals intersecting at numerous levels within a world hierarchy of urbanization and that, within any comprehensive description of our communities, non-spatial structures of social, economic, and institutional activity realms must become as important as our past notions of geographic places.(34)

Exploring in depth the process of creative environmental design, Alexander has proposed a systematic methodology for planners that requires

explicit a priori specification of all decision criteria and, where appropriate, exploits the neutrality of the computer in organizing with respect to stated values the planner's progression from problem analysis to solution synthesis.[35] Within the scope of current efforts to model via computer simulation the sensitivity of community land developments patterns to public policy-making, and hence to enhance the rationality of public decision-making with respect to future urban environments,[36] the present author has pursued mathematical programming techniques that would allow a computer-assisted search for future urban land use-transportation alternatives maximally congruent with projected social activity and communication systems.[37,38,39] It was in this context that the above pattern correlation measures were developed.

To date, the primary motivation for pattern information processing research at CAC has been a cooperative commitment with the Laboratory for Applications of Remote Sensing (LARS) of Purdue University to explore applications of the ILLIAC IV parallel computer[40] at NASA's Ames Research Center and the nationwide ARPA Network[41] in support of the earth resources monitoring objectives of the ERTS/EROS programs of NASA and the U. S. Department of Interior.[42] Multispectral image processing software previously researched at LARS has been redeveloped in parallel computation mode to allow evaluation of the potential efficiencies of ILLIAC IV interpretation of large quantities of natural resource, agricultural, and urban land use data now obtainable by current remote sensing technologies such as the Earth Resources Technology Satellite (ERTS). Additionally, we are exploring the practicalities of computer communications networking as a mechanism to insure decentralized access by government resource management

agencies to whatever centralized data processing facilities may prove efficient. We hope to explore the applicability of the pattern correlation techniques presented above to a number of data processing tasks within this research.

Throughout all of this work, we find ourselves living Wiener's thesis of 1950:

> "... that society can only be understood through a study of the messages and the communication facilities which belong to it; and that in the future development of these messages and communication facilities, messages between man and machines, between machines and man, and between machine and machine, are destined to play an ever-increasing part." (3)

The mechanical information processing and communication technologies in service to man today rest firmly on the foundations of encoded information theory established by Wiener and Shannon more than two and a half decades ago. Much of our current pattern information processing research is motivated by a continued enthusiasm to comprehend better the nature of human communication processes in order to be able to program the machine to simulate--in some primitive fashion--understanding of man's own communications and thus to make convenient the machine's subservience to man on man's own terms. A more important outcome of this research, we think, will be a more complete appreciation for the unique characteristics and capacities of human communications and an acute recognition of the primacy of human communications for the continued realization of the potentialities of man.

From the standpoint of modern psychology, the present model of pattern communications seems promising in that it suggests possible new

approaches for development of macro mathematical theories of the information encoding, transmission, and decoding processes that take place within our own brains and hence control the complex organization of pattern information we experience as language, thought, and awareness.

Since our model defines patterns as distributions of quantized information over sets of discrete, information storage elements, by analogy, we may speculate the pattern information of the brain to exist as pattern-specific biochemical substances distributed throughout intricate networks of neurons. Such a conceptualization of information within the brain would seem apt not only for representation of afferent and efferent memory traces within the modal centers of the brain, but also for representation of complex internuncial associations of patterns within and between modal areas throughout highly structured pathways as well as random cortical networks.

The possibility that the processes of information organization within the brain that give rise to language and thought, however parallel and complex, might be scientifically demonstrated to be governed by some dynamic compromise between the natural laws of energy conservation and entropy maximization--concepts so fundamental to our present understanding of the physical world around us and becoming increasingly important for our description of our social environment--is indeed exciting. Such a demonstration should contribute greatly toward man's confidence in the uniqueness within nature of his own capacity for information acquisition, assimilation, and organization--pattern synthesis--a capacity then mathematically expressible only as some joint product of the factorial complexity of possible synaptic transmissions and the enormous capacities

for information coding particular biochemical substances within the brain are known to process.[43,44,45]

Referencing continually throughout his own work the ideas of men such as Leibniz, Locke, and Bergson, Wiener viewed our present paradigm of cybernetics simply as a logical continuation of man's universal inquiry into the nature of his own being and his destiny. Summarizing a succession of individual perspectives on human communication, Matson and Montagu have more recently termed this inquiry the "unfinished revolution" of communications.[46] That present communication technologies now make practical for this inquiry a platform of planetary dimensions, and that our theories of community and communications continue to develop in harmony, perhaps both as extensions of a natural psychology of man, give us much hope that this inquiry will be carried into the future by and for a global order of man, an order continually rising above a diversity of cultural languages and national institutions. As we seek for our own present generation participation within this social dialogue, we must proceed with a reverence for human communication similar to that expressed by Dewey:

> "Communication is uniquely instrumental and uniquely final. It is instrumental as liberating us from the otherwise overwhelming pressure of events and enabling us to live in a world of things that have meaning. It is final as a sharing in the objects and arts precious to a community, a sharing whereby meanings are enhanced, deepened and solidified in the sense of communion...When the instrumental and final functions of communication live together in experience, there exists an intelligence which is the method and reward of the common life, and a society worthy to command affection, admiration, and loyalty."[47]

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